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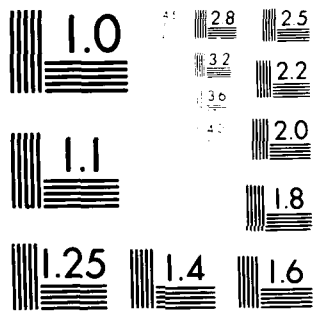
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based on the data queue length is employed to cutoff the priority of voice to prevent a data queue buildup. A continuous-time queueing model was developed and the performance of the flow control scheme was obtained using an efficient computational procedure.
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A STUDY OF INTEGRATED SWITCHING TECHNIQUES

by

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1. INTRODUCTION

Circuit switching is used widely for voice communication and packet switching has been successfully demonstrated and implemented for data communications. Both of these services are frequently provided by separate networks. There has been a recent emphasis on voice transmission and processing in the digital form [1,2]. This has led to an important issue in telecommunication network design as to whether circuit switching and packet switching be provided by separate networks or they can be combined in an efficient fashion into a single network. An integrated approach to the switching of voice and data is attractive because it provides a more cost-effective utilization of communication resources, e.g., transmission and switching facilities. In addition, it is capable of accomodating a wide variety of traffic types which are anticipated both in civilian and military telecommunication environments, e.g.,

- (i) a wide range of traffic rates from low speed teletype terminals requiring hundreds of bits/sec to wide band video and graphics requiring hundreds of kilobits/sec.
- (ii) a wide range of transaction sizes from short interactive messages requiring several hundred bits to bulk data transfers of millions of bits.
- (iii) a wide range of delivery time requirements ranging from continuous, near real time requirements of voice and video to intermittent operation of interactive or bulk data users which may be queued.

Integrated switching also provides the possibility of interconnecting a broad community of user terminals. These considerations have motivated the telecommunication system planners and designers to investigate the concept of integrated switching and implementation of integrated communication networks. This current

interest is evidenced by a number of articles in the recent special issue of military communication of the IEEE Transactions on Communications [3,4,5].

During the research grant period, we dealt with an analytical study of integrated switching techniques. Two problems have been considered. First, we considered the modeling and analysis of integrated multiplexing of voice and data. We have developed two discrete-time queueing models for integrated multiplexers of voice and data, one uses silence activity detection and the other does not. In these models, the amount of computational effort required to obtain numerical results in realistic situations is very large and, therefore, approximations to the models have been considered. In Appendix A, we provide a summary of our work on this problem. Secondly, we considered the flow control problem for a movable boundary integrated voice-data multiplexer. A flow control scheme has been proposed where a decision rule based on the data queue length is employed to cutoff the priority of voice to prevent a data queue build up. A continuous-time queueing model for the integrated multiplexer has been developed and the performance of the flow control scheme has been obtained using an efficient computational procedure. Details of this work are provided in Appendix B.

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MODELING AND ANALYSIS OF INTEGRATED VOICE-DATA MULTIPLEXERS

A.1 Introduction

The concept of integrated switching which combines the features of circuit switching and packet switching has been introduced quite recently [1-3] and is, in essence, a hybrid scheme. An integrated switching facility provides both packet and circuit switching service in that it supports a user community requiring either circuit switched service (e.g. telephones) or packet switched service (e.g. interactive traffic) or both. The high bandwidth transmission capacity is shared by the two types of traffic. The scheme is implemented in terms of a synchronous time-division multiplexed (TDM) frame structure. The channel is synchronously clocked and is divided into frames of fixed duration. Each frame is further subdivided into slots. The frame and slot durations are determined by the voice digitization rate and the transmission rate. Circuit switched voice requires a single slot for transmission whereas packet switched data may require a multiple number of slots for transmission. The TDM frame is partitioned into two regions by means of a boundary. One region is dedicated to circuit switched traffic and the other to packet switched traffic. All circuit switched traffic which cannot seize channel capacity upon arrival is blocked. All packet switched traffic which cannot be transmitted is buffered and is serviced on a first-come-first-served basis. The boundary which partitions the TDM frame may be fixed which implies that there is no dynamic sharing of channel capacity. The boundary can also be movable which allows for the packet switched traffic to utilize any idle circuit switched slots. This movable boundary case is also known as the dynamic channel allocation scheme [4,5]. The movable boundary implementation is better due to the dynamic sharing of the channel capacity between voice and data. Refinements of the basic integrated multiplexing scheme have been considered in [6,7].

Modeling and analysis of integrated switching was first attempted by Kümmerle [2] where a very approximate analysis was performed. Fischer and Harris [8] presented a more accurate mathematical model for evaluating the performance of an integrated circuit-and packet-switched multiplexer. However, in their queueing analysis of the data traffic, the Markov dependence between the number of voice calls in adjacent frames was overlooked. Our mathematical model, that we shall present in the next section takes this dependence into account. A continuous-time analysis of integrated multiplexers has been performed by Chang [9]. This, however, is an approximation since the actual system has a frame structure. Our model is a discrete-time model which is more appropriate for integrated multiplexers. Modeling and simulation of these systems is also considered in [10]. Next, we present a brief summary of our work on the modeling and analysis of integrated multiplexers.

A.2 Analytical Results

As indicated previously, we have pursued the modeling and analysis of the movable boundary frame allocation scheme. In this scheme, a fixed length frame consisting of $(S + N)$ slots is assumed. N of the slots are reserved for data and the remaining S slots are shared by voice and data with voice having priority over data. Packetized transmission for both voice and data is assumed. To simplify the presentation here, the packet sizes for voice and data are assumed to be identical. Voice traffic operates on a loss basis, while the data traffic is buffered in an infinite buffer and is serviced on a first-come-first-served basis. Voice and data arriving during any frame are considered for service only at the beginning of the next frame. The performance measures for voice traffic and data traffic are probability of loss and expected waiting time respectively.

The voice traffic can be characterized in terms of its on/off (or active/inactive) periods. Both interarrival time and holding time for voice are assumed to be exponentially distributed with means $1/\lambda_1$ and $1/\mu_1$ respectively. Furthermore, each active voice source can be characterized in terms of talkspurt and silence durations which are also assumed to be exponentially distributed with means $1/\eta_1$ and $1/\xi_1$ respectively. We assume that the voice traffic can have at most one on/off transition during any frame. This is a realistic assumption since the frame duration, T , is usually much smaller than the mean interarrival time, $1/\lambda_1$, and mean holding time, $1/\mu_1$. Since the mean talkspurt/silence durations are usually much larger than T , we also assume that there can be at most one talkspurt/silence transition during any frame. The voice traffic can, therefore, be statistically modeled by a pair of Markov chains shown in Figure A.1. The Markov chain shown in Figure A.1(a) is for the number of active voice calls and it describes the system state corresponding to the voice traffic variations due to the initiation and termination of voice

calls. The Markov chain of Figure A.1(b) is for the number of voice calls that are in the talkspurt mode given that K of the maximum allowable S voice calls are active. The interactive data traffic is assumed to follow a Poisson arrival pattern with rate λ_2 and its message length, in terms of number of packets, has a known general distribution with mean $1/\mu_2$.

We have considered two different schemes for the multiplexing of voice and data. The first scheme operates without a speech activity detector (SAD) and does not utilize the silence periods of voice calls. In the second scheme, an SAD is employed so that the silence periods of the voice calls can be detected and utilized for data transmission thereby making the system more efficient. In the following, we present the analysis for both the schemes.

A.2.1 Voice/Data Multiplexer Without SAD:

Let us define the following quantities which correspond to the j th frame:

L_j = Number of data packets in queue at the beginning of the j th frame,

S_j = Number of slots occupied by voice in the j th frame.

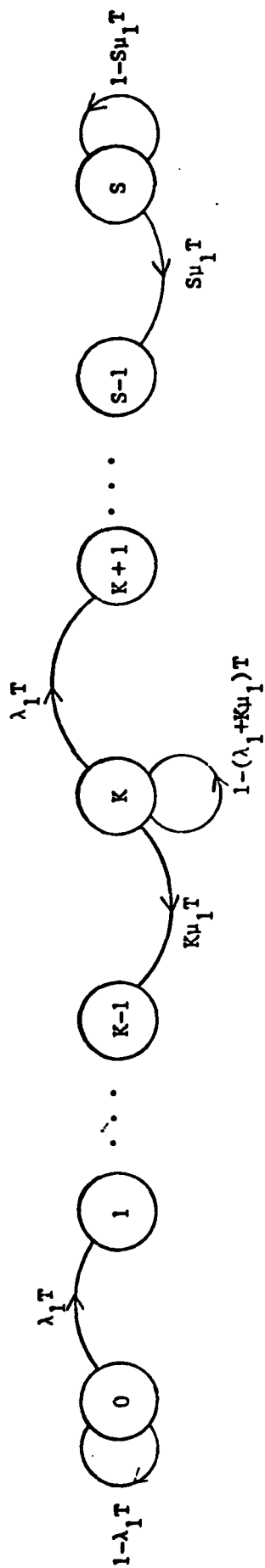
v_{j+1} = Number of data packet arrivals during the j th frame.

In the case of a voice/data multiplexer without SAD, the Markov chain shown in Fig. A.1(a) describes the statistical variations in the number of slots occupied by voice from frame to frame. Using the Markov chain analysis, the steady state probability, p_k , that k slots are occupied by voice in a frame is found to be

$$p_k = \frac{\rho_1^k / k!}{\sum_{i=0}^S \rho_1^i / i!} \quad ; \quad k=0,1,2,\dots,S. \quad (A.1)$$

where $\rho_1 = \lambda_1 / \mu_1$. The loss probability, P_L , for voice equals the probability that all the S slots available for voice are occupied. Hence,

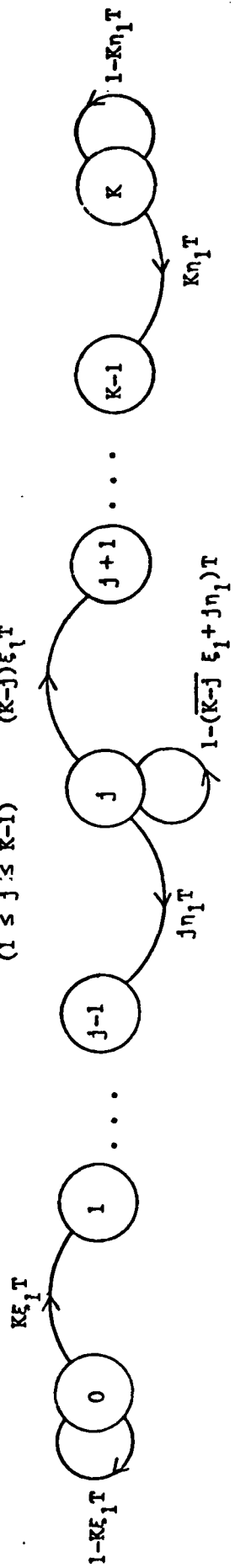
$(1 \leq K \leq S-1)$



(a)

$(1 \leq j \leq K-1)$

$(K-j)\xi_1 T$



(b)

$(K = 0, 1, 2, \dots, S)$

Figure A.1 Markov chains for the characterization of voice traffic.

$$P_L = \frac{\rho_1^S / S!}{\sum_{k=0}^S \rho_1^k / k!} \quad (A.2)$$

For the data queue, we write

$$L_{j+1} = [L_j - (S + N - S_j)]^+ + v_{j+1} \quad (A.3)$$

where $[K]^+ = 0$ if $K < 0$ and $[K]^+ = K$ if $K \geq 0$. The sequence $\{(L_j, S_j)\}$ forms a two-dimensional Markov chain. We define,

$$X_n(z) = \sum_{l=0}^{\infty} P\{L_j=l, S_j=n\} z^l ; \quad n=0,1,2,\dots,S \quad (A.4)$$

where the joint probabilities appearing on the right hand side correspond to the steady state. Then, the moment generating function (m.g.f.) for the data queue, $L(z)$, can be expressed as

$$L(z) = \sum_{n=0}^S X_n(z) \quad (A.5)$$

In order to obtain an expression for $L(z)$, we note the following:

$$P\{L_{j+1}=l, S_{j+1}=n\} = \sum_{k=0}^{\infty} \sum_{m=0}^S P\{L_{j+1}=l, S_{j+1}=n | L_j=k, S_j=m\} \cdot$$

$$P\{L_j=k, S_j=m\}$$

$$= \sum_{k=0}^{\infty} \sum_{m=0}^S P\{L_{j+1}=l | L_j=k, S_j=m\} \cdot P\{S_{j+1}=n | S_j=m\} \cdot P\{L_j=k, S_j=m\}$$

or,
$$P\{L_{j+1}=l, S_{j+1}=n\} = \sum_{k=0}^{\infty} \sum_{m=0}^S P\{v_{j+1} = l - [k - (S + N - m)]^+\} \cdot$$

$$P\{S_{j+1}=n | S_j=m\} \cdot P\{L_j=k, S_j=m\} \quad (A.6)$$

From eqns. (A.4) and (A.6) after some algebraic manipulations, it can be shown that the following matrix equation holds for the vector $\underline{X}(z) = [X_0(z), X_1(z), X_2(z), \dots, X_S(z)]^t$:

$$[I - G(z) \underline{A}(z)] \underline{X}(z) = G(z) \underline{B}(z)$$

(A.7a) 10

where $G(z)$ is the m.g.f. of v_j . The $(S+1) \cdot (S+1)$ elements of the matrix $\underline{A}(z)$ are the following:

$$\begin{aligned} A_{00} &= (1 - \lambda_1 T) z^{-(S+N)} \\ A_{SS} &= (1 - S\mu_1 T) z^{-N} \\ A_{i,i-1} &= \lambda_1 T z^{-(S+N-i-1)} \quad ; \quad i=1,2,\dots,S. \\ A_{i,i} &= (1 - \lambda_1 + i\mu_1 T) z^{-(S+N-i)} \quad ; \quad i=1,2,\dots,S-1. \\ A_{i,i+1} &= (i+1)\mu_1 T z^{-(S+N-i-1)} \quad ; \quad i=0,1,\dots,S-1. \\ A_{i,j} &= 0 \quad \text{for } |i-j| \geq 2 \end{aligned} \quad (A.7b)$$

The $(S+1)$ elements of the vector $\underline{B}(z)$ are the following:

$$\begin{aligned} B_0(z) &= (1 - \lambda_1 T) D_0(z) + \mu_1 T D_1(z) \\ B_S(z) &= \lambda_1 T D_{S-1}(z) + (1 - S\mu_1 T) D_S(z) \\ B_n(z) &= \lambda_1 T D_{n-1}(z) + (1 - \lambda_1 + n\mu_1 T) D_n(z) + (n+1)\mu_1 T D_{n+1}(z) ; \\ & \quad 1 \leq n \leq S-1 \end{aligned} \quad (A.7c)$$

where

$$D_n(z) = \sum_{k=0}^{S+N-n-1} [1 - z^{-(S+N-n)+k}] P\{L_j=k, S_j=n\} \quad (A.7d)$$

Since the data messages are assumed to arrive according to a Poisson process with rate λ_2 it can be shown that

$$G(z) = \exp [-\lambda_2 T (1 - H(z))] \quad (A.8)$$

where $H(z)$ is the m.g.f. of the data message length (If all of the arriving data messages are one packet long, then $H(z)=z$).

It can be shown that the steady state distribution for the data queue length exists, provided that

$$\frac{\lambda_2}{\mu_2} T < S + N - \beta \quad (A.9)$$

where β denotes the expected value of S_j in the steady-state.

The unknown probabilities in $\underline{B}(z)$ are found by using (i) Rouché's theorem, and (ii) the fact that $X(z)$ exists for all values of z within the unit circle.

Then $X(z)$ and $\underline{L}(z)$ are known from eqns.(A.7) and (A.5) respectively. The expected waiting time for the data messages, EW , is then obtained as

$$EW = \frac{T}{2} + \frac{\mu_2}{\lambda_2} \left\{ \frac{dL(z)}{dz} \right\}_{z=1} \quad (A.10)$$

A.2.2 Voice/Data Multiplexer with SAD:

In the case of a voice/data multiplexer with SAD, both of the Markov chains depicted in Fig.A.1 are necessary to describe the statistical variations in the number of slots (in a frame) that are occupied by voice. In addition to L_j , S_j and v_{j+1} defined previously, let us now define R_j to be the number of callers who are 'on' during the j th frame. Then R_j is the state of the Markov chain in Fig. A.1(a) and S_j is the state of the Markov chain in Fig. A.1(b) when $R_j=K$. It should be observed that the triple (L_j, R_j, S_j) forms a Markov chain. With the help of this Markov chain, we can perform exact analysis of the voice/data multiplexer with SAD. However, obtaining numerical values of interest (e.g. data message delay) using the results of such an exact analysis is computationally not feasible. Based on certain characteristics of the voice and data traffics encountered in practice, here we present an approximate analysis of the voice/data multiplexer with SAD. Even with this approximation, computations involved in obtaining numerical values are considerably large for practical values of S and N . Approximations which simplify the analysis, and ease computations even further have been considered and are discussed in the Section A.3.

Since the data traffic is assumed to be of the interactive type, the data messages will have service durations that are almost always much smaller than the on or off times of the voice traffic. Hence, following each on/off

transition of the voice traffic, the data traffic queue behavior reaches a steady state in a time that is almost always much smaller than the time till the next occurrence of an on/off transition for voice. Hence we may, as a good approximation, analyze the data traffic performance with a fixed $R_j = K$, and find the corresponding expected waiting time, EW_K , for data with S_j varying according to the talkspurt/silence Markov model of Fig. A.1. Then to obtain the overall expected waiting time for data, the $(S+1)$ waiting times, EW_K ; $K = 0, 1, 2, \dots, S$, are averaged.

In the present model, R_j is same as the S_j of Section A.2.1. Hence the steady state probabilities of R_j are given by A.1. The expression for the loss probability remains the same as that in the case of a voice/data multiplexer without SAD. This is so because the silence periods of voice are used to provide increased service to data alone; call initiation requests are treated just as in the case of a voice/data multiplexer without SAD. Analysis of the Markov chain in Fig. A.1(b) reveals that the steady state probabilities of S_j , the number of slots occupied by voice packets in a frame (for R_j fixed at K), are given by

$$\pi_{K,k} = \frac{\binom{K}{k} \gamma_1^k}{\sum_{j=0}^K \binom{K}{j} \gamma_1^j} \quad ; \quad \begin{array}{l} k=0,1,2,\dots,K \\ K=0,1,2,\dots,S \end{array} \quad (A.11)$$

where $\gamma_1 = \xi_1/\eta_1$

For the data queue we have,

$$L_{j+1} = [L_j - (S + N - S_j)]^+ + v_{j+1} \quad (A.12)$$

wherein we assume that R_j is fixed at K . Noting that the pair (L_j, S_j) forms a two-dimensional Markov chain, we define

$$X_n(z) = \sum_{l=0}^{\infty} P \{L_j=l, S_j=n\} \quad ; \quad n=0,1,2,\dots,K \quad (A.13)$$

Then the m.g.f. of the data queue length, $L(z)$ can be expressed as

$$L(z) = \sum_{n=0}^K X_n(z) \quad (A.14)$$

The analysis for the data queue here is similar to that given in Section A.2.1. It follows that the vector $X(z) = (X_0(z), X_1(z), \dots, X_K(z))^T$ is given by the following matrix equation

$$[I - G(z) \quad A(z)] \underline{X}(z) = G(z) \underline{B}(z) \quad (A.15a)$$

where $G(z)$ as described earlier, is the m.g.f. of v_j . The elements of the $(K+1) \times (K+1)$ matrix $A(z)$ are the following:

$$\begin{aligned} A_{00} &= (1 - K\xi_1 T) z^{-(S+N)} \\ A_{KK} &= (1 - K\eta_1 T) z^{-(S+N-K)} \\ A_{i,i-1} &= (K-i-1)\xi_1 T z^{-(S+N-i-1)} \quad ; \quad i=1,2,\dots,K \\ A_{i,i} &= [1 - (K-i)\xi_1 + i\eta_1] T z^{-(S+N-i)} \quad ; \quad i=1,2,\dots,K-1 \\ A_{i,i+1} &= (i+1)\eta_1 T z^{-(S+N-i+1)} \quad , \quad i=0,1,2,\dots,K-1 \\ A_{i,j} &= 0 \quad \text{for} \quad |i-j| \geq 2. \end{aligned} \quad (A.15b)$$

The $(K+1)$ elements of $\underline{B}(z)$ in eqn. (A.15a) are the following:

$$\begin{aligned} B_0(z) &= (1 - K\xi_1 T) D_0(z) + \eta_1 T D_1(z) \\ B_K(z) &= \xi_1 T D_{K-1}(z) + (1 - K\eta_1 T) D_K(z) \\ B_n(z) &= (K-n-1)\xi_1 T D_{n-1}(z) + [1 - (K-n)\xi_1 + n\eta_1] T D_n(z) \\ &\quad + (n+1)\eta_1 T D_{n+1}(z) \quad ; \quad 1 \leq n \leq K-1 \end{aligned} \quad (A.15c)$$

where $D_n(z)$ is as given by (A.7d). For steady state probabilities of the data queue length to exist, it can be shown that the following inequality must be satisfied:

$$\frac{\lambda_2}{\mu_2} T < S + N - E_K[S_j] \quad (A.16)$$

where $E_K[S_j]$ is the steady state expected value of S_j while R_j is fixed at K . Now, from (A.14) and (A.15), the moment generating function, $L(z)$, for the data queue (corresponding to R_j fixed at K), can be obtained as explained before in Section A.2.1. The expected waiting time, EW_K , corresponding to R_j fixed at K is then given by

$$EW_K = \frac{T}{2} + \frac{\mu_2}{\lambda_2} \left\{ \frac{dL(z)}{dz} \Big|_{z=1} \right\} \quad (A.17)$$

The queueing analysis described above is performed $(S+1)$ times corresponding to all values of K in the range 0 to S . The stability condition of (A.16) must then be satisfied for each value of K in the range 0 to S . The mean $E_K[S_j]$ appearing in (A.16) is the mean of the probability distribution of (A.11). It can be shown that $E_K[S_j]$ monotonically increases with K . Hence, the approximate analysis under consideration requires the following condition for stability:

$$\frac{\lambda_2}{\mu_2} T < S + N - E_S[S_j] \quad (A.18)$$

The above condition is more strict than what is actually required for the stability of the data queue length. It can be shown that the following is a necessary and sufficient condition for the stability of the data queue:

$$\frac{\lambda_2}{\mu_2} T < S + N - E[S_j] \quad (A.19)$$

where $E[S_j]$ is the steady state expected value of S_j . The expression to the right of the inequality sign above is the expected number of slots available per frame to data traffic. Due to the assumptions made earlier regarding the transitions of R_j and S_j , it follows that the following approximate relation holds for $E[S_j]$:

$$E[S_j] = \sum_{K=0}^S p_K E_K[S_j] \quad (A.20)$$

Provided (A.18) is true, the overall expected waiting time, EW , for the data traffic messages is to be computed by averaging $\{EW_K; K=0,1,\dots,S\}$:

$$EW = \sum_{K=0}^S p_K EW_K \quad (A.21)$$

A.3 Approximations and Numerical Results

The analytical results presented in Section A.2 were based on a Markov chain model for the voice traffic. Such a Markov chain model for the voice traffic is quite accurate [11, 12]. Consequently, the analysis of the data queue based on the Markov chain model for the voice traffic gives practically exact results. However, these exact analytical results require tremendous computational effort in finding numerical values of interest concerning the data queue statistics. For example, finding the expected data queue length using the results of Section A.2.1 involves (i) finding $[(S + 1)(S + N - \frac{S}{2}) - 1]$ complex roots of an analytic function, and (ii) solving as many complex linear simultaneous equations. In practice, the values of S and N are quite large, and, therefore, a large computational effort is required. All the above complex roots are known to lie within the unit circle. There is a one-to-one correspondence between the roots and the simultaneous equations. The relationship is such that, while solving the simultaneous equations, numerical singularity problems may arise if any two or more of the roots lie too close to each other. In this section, therefore, we consider some approximations to the voice traffic model. The purpose of these approximations is to reduce the computational effort involved in determining the data queue statistics. The analytical results corresponding to each of these approximations are derived and a numerical example is presented. The validity of these approximations is also discussed.

(1) I.I.D. Approximation

Let S_j denote the number of slots occupied by voice in the j th frame. In the i.i.d. approximation, we assume that $\{S_j; j=1,2,\dots\}$ is an independent and identically distributed (i.i.d.) random sequence. In Section A.2, we derived the steady state probabilities for S_j using the Markov chain analysis. We assume that the same probabilities apply in the present approximation for the i.i.d. sequence $\{S_j; j=1,2,\dots\}$. This approximation is the same as that considered by Fischer and Harris [8]. Let L_j and v_j be as defined in Section

A.2.1... The steady state moment generating function (m.g.f.) for the data queue length is defined as

$$L(z) = \sum_{k=0}^{\infty} \pi_k z^k \quad (A.22)$$

where $\pi_k \stackrel{\Delta}{=} P\{L_j=k\}$ is the steady state probability that there are k packets in the data queue at the beginning of a frame. Using the m.g.f. approach it can be shown that $L(z)$ can be expressed as

$$L(z) = \frac{G(z) \sum_{m=0}^S p_m \sum_{k=0}^{S+N-1-m} (z^{S+N} - z^{m+k}) \pi_k}{z^{S+N} - G(z) V(z)} \quad (A.23)$$

where $S, N, \{p_m\}$, and $G(z)$ are the same as those defined in Section A.2. The function $V(z)$ denotes the m.g.f. for S_j , i.e.,

$$V(z) = \sum_{m=0}^S p_m z^m \quad (A.24)$$

The expression for $L(z)$ given in eqn. (A.23) contains $(S+N)$ unknown probabilities $\{\pi_k; k=0,1,\dots, S+N-1\}$. The procedure for finding these unknown probabilities is well known in queueing theory. Once these probabilities are determined, $L(z)$ is completely defined. Then eqn. (A.10) is used to find the expected waiting time for the data messages.

(ii) Mean-Value Approximation

Let β denote the steady state expected value of S_j based on the Markov chain analysis of Section A.2.1. It may be noted that β will not be an integer quantity in general. Let K denote the largest integer less than or equal to β . In the mean-value approximation, we assume that the sequence $\{S_j; j=1,2,\dots\}$ is i.i.d. with state space $(K, K+1)$. The probability distribution for this i.i.d. sequence is so chosen that the expected value of S_j remains β .

Thus we have

$$\begin{aligned}\theta_K &\stackrel{\Delta}{=} P\{S_j=K\} = K+1-\beta \\ \theta_{K+1} &\stackrel{\Delta}{=} P\{S_j=K+1\} = \beta-K\end{aligned}\quad (A.25)$$

Based on the mean-value approximation, the expression for the m.g.f. of the data queue length is shown to be

$$L(z) = \frac{G(z) \sum_{m=K}^{K+1} \theta_m \sum_{k=0}^{S+N-1-m} (z^{S+N-K} - z^{m-K+k}) \pi_k}{z^{S+N-K} - G(z) U(z)} \quad (A.26)$$

where $U(z)$ is given by

$$U(z) = \theta_K + \theta_{K+1} z \quad (A.27)$$

If β is an integer, then $K=\beta$ and hence $\theta_K=1$, $\theta_{K+1}=0$. The expression for $L(z)$ is then given by

$$L(z) = \frac{G(z) \sum_{k=0}^{S+N-K-1} (z^{S+N-K} - z^k) \pi_k}{z^{S+N-K} - G(z)} \quad (A.28)$$

There are $(S+N-K)$ unknown probabilities $\{\pi_k; k=0,1,2,\dots, S+N-K-1\}$ to be determined to completely define $L(z)$ under the mean-value approximation.

(iii) Slow-Variation Approximation

Usually, the mean interarrival and holding times for the voice traffic are much longer than those for the data traffic. As a result, S_j remains constant over a long period consisting of many frames while the data queue reaches a steady state quickly and operates in the steady state for most of that period. The slow variation approximation is based on this observation. We assume that a constant number, say m , of the slots are occupied by voice in each frame, and perform a steady state analysis of the data queue. The analysis is performed for values of m ranging from 0 to S . For a given m , the expression

for the data queue m.g.f. is given by

$$L_m(z) = \frac{G(z) \sum_{k=0}^{S+N-m-1} (z^{S+N-m} - z^k) \pi_m(k)}{z^{S+N-m} - G(z)} \quad (\text{A.29})$$

where $\pi_m(k) \triangleq P\{L_j=k | (S_i=m, \text{ for all } i)\}$. Let W_m denote the expected waiting time for the data queue when exactly m slots are occupied by voice in each frame. The overall expected waiting time EW is obtained by averaging $\{W_m\}$ as follows

$$EW = \sum_{m=0}^S p_m W_m \quad (\text{A.30})$$

where $\{p_m\}$ are the steady state probabilities ($P\{S_j=m\}$; $m=0,1,\dots,S$) as derived in Section A.2.1.

The slow variation approximation is valid only if the data queue described by eqn. (A.29) is stable for each m . Thus the stability condition for the slow-variation approximation is given by

$$\frac{\lambda_2}{\mu_2} T < N \quad (\text{A.31})$$

where λ_2 is the data message arrival rate, μ_2 is the mean data message length in packets and T is the frame duration.

Numerical Example:

In the numerical example presented here, we choose the following values for the system parameters:

$$S = 10, N = 5$$

$$\lambda_1 = 0.05 \text{ sec}^{-1} \quad (\text{voice arrival rate})$$

$$\frac{1}{\mu_1} = 100 \text{ sec} \quad (\text{voice holding time})$$

We assume that the data messages are of fixed duration, each requiring one slot for service. Based on each approximation we compute the expected message delay, EW , for the data traffic for various values of $\lambda_2 T$, the expected number of data packet arrivals per frame. Plots of EW vs $\lambda_2 T$ are shown in Fig. A.2 for the different approximations.

Discussion of Results:

Some comparisons between the different approximations and the exact analytical results are listed in Table A.1. In order to obtain numerical results, in each approach some complex roots of an analytic function are to be determined and then a set of complex linear simultaneous equations are to be solved. It can be observed that the approximations require much less computations than the exact analysis. The stability condition for the i.i.d. and mean-value approximations is the same as that of the actual system. However, the slow-variation approximation requires a stricter stability condition. The expected message delays based on the approximations do not compare well with the simulation results of Weinstein et al [10] in the range $\lambda_2 T \geq N$. Long periods of high channel occupancy by voice cause excessive data packet delays [10, 13]. These excessive delays are not well accounted for in the approximations considered here. However, the slow variation approximation is expected to give satisfactory results in the range $\lambda_2 T < N$. This is because the slow variation approximation closely represents the actual voice traffic characteristics.

	# roots/ # simult. eqns.	Stability Condition	Comments
I.I.D.	C-1	$\frac{\lambda_2^T}{\mu_2} < C-\beta$	
Mean-Value	C-K	$\frac{\lambda_2^T}{\mu_2} < C-\beta$	Most optimistic
Slow-Variation	$[(S+1) \cdot (C - \frac{S}{2}) - 1]^*$	$\frac{\lambda_2^T}{\mu_2} < N$	most conserva- tive, stricter stability condition
Exact Discrete- Time	$[(S+1) \cdot (C - \frac{S}{2}) - 1]^+$	$\frac{\lambda_2^T}{\mu_2} < C-\beta$	

* involves no matrix operations

† involves matrix operations

Table A.1. Some comparisons between the different models.

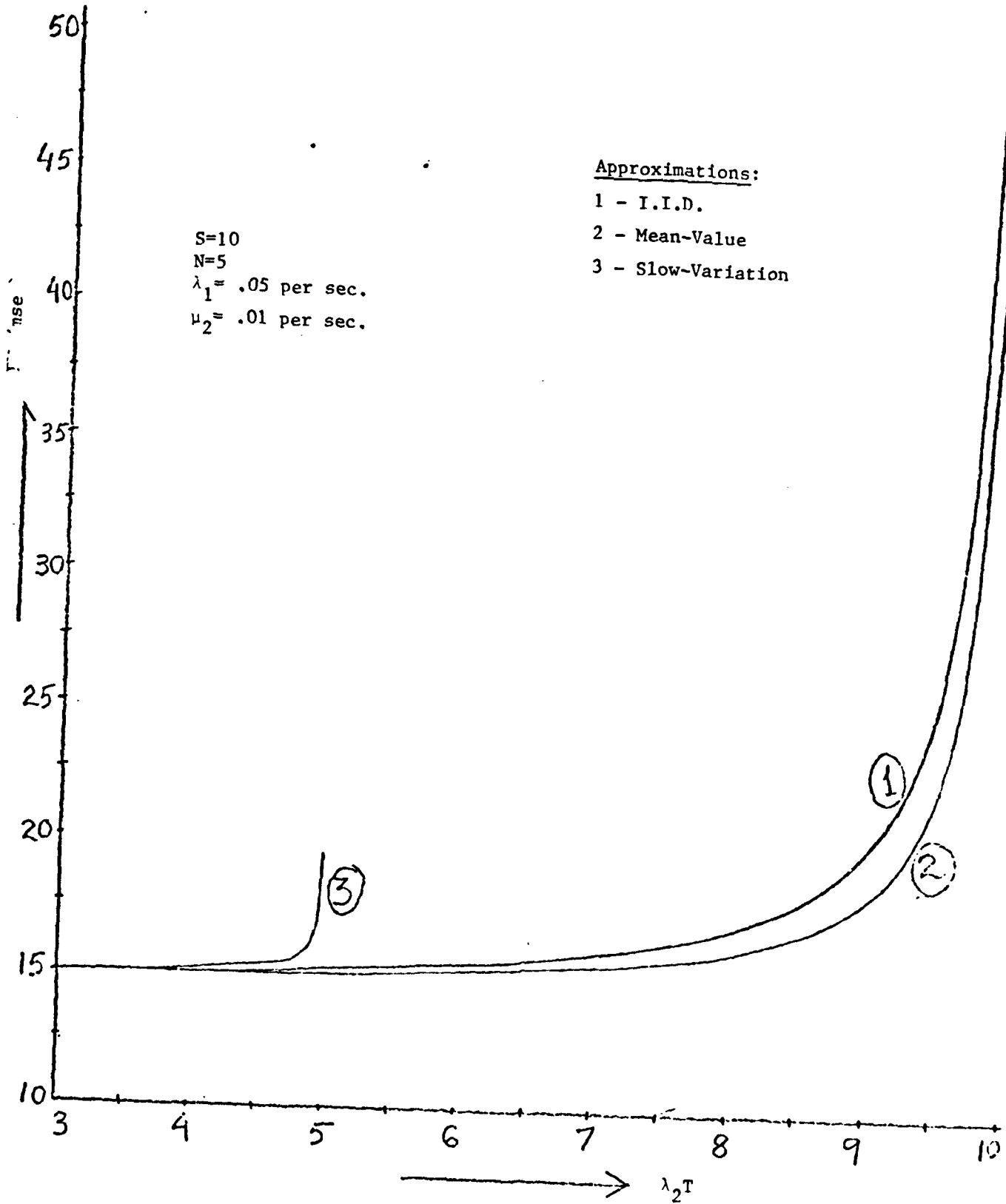


Fig. A.2. Mean data packet delay (FW) vs mean packet arrivals per frame ($\lambda_2 T$).

A.4 References

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PRIORITY CUTOFF FLOW CONTROL SCHEME FOR INTEGRATED VOICE-DATA MULTIPLEXERS

B.1 Introduction

Circuit switching is used widely for voice communication and packet switching has been successfully demonstrated and implemented for data communications. Both of these services are frequently provided by separate communication networks. Recently, there has been an emphasis on voice transmission and processing in the digital form [1, 2]. This has motivated the current interest in integrated communications where the features of circuit switching and packet switching are combined in an efficient manner. This hybrid scheme, where the high bandwidth transmission capacity is shared by voice and data, has been introduced recently [3, 4] and will be discussed in detail in the next section. Such an integrated approach to the multiplexing of voice and data is attractive because it provides a more cost-effective utilization of communication resources, e.g., transmission and switching facilities. In addition, it is capable of accomodating a wide variety of traffic types and thus provides the possibility of interconnecting a broad community of subscribers.

Modeling and analysis of integrated multiplexers for voice and data has been considered in [3, 5- 8]. Kummerle [3] performed a very approximate analysis of the integrated multiplexer. Fischer and Harris [5] presented a more accurate mathematical model for evaluating the performance of an integrated multiplexer. However, in their queueing analysis of the data traffic, the Markov dependence between the number of voice calls in adjacent frames was over-

looked. Chang [6] performed a continuous-time analysis of integrated multiplexers which is an approximation since the actual system has a frame structure. We have recently considered a discrete-time model for integrated multiplexing of voice and data where the Markov dependence between the number of voice calls in adjacent frames has been taken into account [7]. These analyses and the simulation results presented in [8] have indicated the need for flow control techniques without which excessive data queues may build up. Some flow control techniques and the associated simulation results are discussed in [8, 9]. In [8], two flow control schemes are considered. In the first, voice rate control is implemented without any data flow control. The second one is a combined voice and data flow control scheme. In [9], a packet voice flow control concept based on embedded speech coding is discussed.

In this paper, we propose another flow control scheme for integrated multiplexing of voice and data. This flow control mechanism is based upon a modified cut off priority concept which was considered in [10]. In our scheme, a decision rule based on the data queue length is employed to cut off the priority of voice to prevent a data queue buildup. The scheme proposed in this paper, in addition to providing flow control, provides a continuum of system operating points and, in fact, performs better than the integrated multiplexing techniques considered thus far. In the next section, we discuss the implementation of integrated multiplexing schemes and introduce the problem considered here. In Section three, we present the queue-

ing model for the flow control scheme. The performance of the scheme is examined in section four where some numerical results are also presented. Finally, the results are summarized in the last section.

B.2 Integrated Multiplexing of Voice and Data

An integrated multiplexer supports a user community consisting of both voice and data subscribers. The high bandwidth transmission capacity is shared by the two types of traffic. The scheme is implemented in terms of a synchronous time-division multiplexed (TDM) frame structure. The channel is synchronously clocked and is divided into frames of fixed duration. Each frame is further subdivided into slots. The frame and slot durations are determined by the voice digitization rate and the transmission rate. The TDM frame is shared by voice and data and is partitioned into two regions by means of a boundary, one to be used by voice and the other by data. This boundary may be fixed which implies that there is no dynamic sharing of the channel capacity. This scheme is known as the fixed boundary scheme. The boundary can also be movable which allows for the data traffic to utilize any idle voice slots. This is known as the movable boundary scheme or the dynamic channel allocation scheme [11]. The movable boundary implementation is better due to the dynamic sharing of the channel capacity between voice and data. In order to prevent voice from taking over the system completely, a limit S is placed on the number of slots that the voice can use in a frame. This guarantees a minimum grade of service for data. Any voice call which cannot seize channel capacity upon arrival is blocked, and a data call which cannot be transmitted upon arrival is buffered and is serviced on a first-come-first-served

basis. The movable boundary scheme emphasizes system utilization. The optimal slot allocation problem for a fixed frame has been considered in [12] where the problem is formulated as a Markov decision process and two optimality criteria are considered.

In this paper, we consider the flow control of the movable boundary integrated multiplexing scheme where the maximum number of slots that the voice can use is S . In our flow control scheme, whenever a data queue build up is detected, the priority of voice traffic is reduced and the grade of service for data traffic is improved. A decision rule based on the data queue length is employed to vary the parameter S . If the data queue length exceeds a certain threshold, S may be reduced to S' and accordingly any incoming voice calls will be blocked. It should be noted, however, that a continuation of service to the voice calls already in the system must be provided. In the next section, we further explain the flow control scheme and present a queueing model for it.

B.3 The Queueing Model

In this paper, we consider a continuous-time queueing model for the integrated multiplexer. This is an approximation since the actual system has a frame structure. It, however, has been used in the literature (e.g., [6]) and simplifies the analysis considerably. We consider the queueing model of C parallel servers (C being the total number of slots in a TDM frame) to which the arrivals form two independent Poisson streams, one representing the voice calls with rate λ_0 and the other representing the data calls with rate λ . The lengths of voice and data calls are assumed to be exponentially distributed with means $1/\mu_0$ and $1/\mu$, respectively. If an incoming voice call observes more than $S-1$ voice calls in progress, or more than $S'-1$ ($S' < S$) voice calls in progress and more than $L-1$ data calls in the system, the voice call is lost. Otherwise, this incoming voice call is assigned a server (slot) according to a preemptive scheme over the data calls. Incoming data calls are, however, stored in the buffer until a server becomes available at which time they will go into service on a first-come-first-served basis. It is assumed that the buffer capacity is infinite.

Since the arrivals are assumed to form Poisson streams and service times have exponential distributions, the number of calls in the system can be represented by a two dimensional continuous-time parameter Markov chain on the state space $\{(i,j), i \geq 0, S \geq j \geq 0\}$, where i and j represents the number of data and voice calls, re-

spectively. The infinitesimal generator Q of this Markov chain is a block tridiagonal matrix of the form,

$$Q = \begin{pmatrix} T_0 - \lambda I & \lambda I & & & \\ T_1^0 & T_1 - T_1^0 - \lambda I & \lambda I & & \\ & T_2^0 & T_2 - T_2^0 - \lambda I & & \\ & & & \ddots & \\ & & & & T_{j-1}^0 & T_{j-1} - T_{j-1}^0 - \lambda I & \lambda I \\ & & & & T_j^0 & T_j - T_j^0 - \lambda I & \lambda I \\ & & & & & \ddots & \\ & & & & & & \ddots & \\ & & & & & & & \ddots & \end{pmatrix}$$

where

$$T_i = \begin{pmatrix} -\lambda_0 & \lambda_0 & 0 & \cdot & \cdot & 0 & 0 & 0 \\ \mu_0 & -(\mu_0 + \lambda_0) & \lambda_0 & \cdot & \cdot & 0 & 0 & 0 \\ 0 & 2\lambda_0 & -(2\mu_0 + \lambda_0) & \cdot & \cdot & 0 & 0 & 0 \\ 0 & 0 & 3\mu_0 & \cdot & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & -((S-2)\mu_0 + \lambda_0) \lambda_0 & 0 & \\ 0 & 0 & 0 & \cdot & \cdot & (S-1)\mu_0 - ((S-1)\mu_0 + \lambda_0) \lambda_0 & & \\ 0 & 0 & 0 & \cdot & \cdot & 0 & S\mu_0 & -S\mu_0 \end{pmatrix}, \quad 0 \leq i < L$$

$$T_i = \begin{vmatrix} -\lambda_0 & \lambda_0 & 0 & . & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_0 & -(\mu_0 + \lambda_0) & \lambda_0 & . & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\mu_0 & -(2\mu_0 + \lambda_0) & . & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3\mu_0 & . & 0 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & (S-1)\mu_0 & -((S-1)\mu_0 + \lambda_0) & \lambda_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 & S'\mu_0 & -S'\mu_0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & 0 & 0 & S\mu_0 & -S\mu_0 \end{vmatrix} \quad i \geq 1$$

and T_i^0 , $i = 1, 2, \dots$, are all diagonal matrices with the diagonal entries $\{T_i^0\}_{jj}$, $j = 1, 2, \dots, S+1$, given by

$$\{T_i^0\}_{jj} = \begin{cases} i\mu, & j = 1, 2, \dots, S+1; \quad i = 1, 2, \dots, C-S-1, \\ \begin{matrix} i\mu & j = 1, 2, \dots, C-1 \\ (C-j+1)\mu, & j = C-i+1, \dots, S+1 \end{matrix} & i = C-S, \dots, C-1 \\ (C-j+1)\mu, & j = 1, 2, \dots, S+1; \quad i \geq C \end{cases}$$

Let x_{ij} be the steady state probability (assuming its existence) that there are i data and j voice calls present in the system. Let $\bar{x}_i = (x_{ij}, j = 0, 1, \dots, S)$ and $\bar{x} = (x_0, x_1, \dots)$. Then, the system of linear equations for \bar{x} in matrix form is,

$$\chi^0 = 0$$

with the normalizing condition

$$\chi^e = 1 ,$$

where $e = (1, 1, \dots, 1)^1$ is of appropriate dimension. In a partitioned form, the above equations are,

$$\chi_0(T_0 - \lambda I) + \chi_1 T_1^0 = 0$$

$$\lambda \chi_{i-1} I + \chi_i (T_i - T_i^0 - \lambda I) + \chi_{i+1} T_{i+1}^0 = 0 , \quad i \geq 1 .$$

Since $T_i^0 = T_C^0$, $i \geq C$ and $T_i \triangleq T'$, $i \geq L$, we have

$$T_i - T_i^0 = T' - T_C^0 , \quad i \geq \max(C, L) \triangleq K .$$

So the above set of equations can be rewritten as

$$\chi_0(T_0 - \lambda I) + \chi_1 T_1^0 = 0 , \tag{B.1}$$

$$\lambda \chi_{i-1} I + \chi_i (T_i - T_i^0 - \lambda I) + \chi_{i+1} T_{i+1}^0 = 0 , \quad K > i \geq 1 , \tag{B.2}$$

$$\lambda \chi_{i-1} I + \chi_i (T' - T_C^0 - \lambda I) + \chi_{i+1} T_{i+1}^0 = 0 , \quad i \geq K . \tag{B.3}$$

Markov chains of this type have been studied in great detail by Neuts [13 - 17]. A similar formulation is carried out by Feldman and Claybough [18] for a queueing model of an integrated voice-data multiplexer with non-preemptive priority. The steady state probability vector \bar{x} is shown to have a modified matrix-geometric form. That is, it can be shown that (easily verified by substitution, see [13 - 17]),

$$\bar{x}_i = \bar{x}_{K-1} R^{i+1-K}, \quad i \geq K, \quad (B.4)$$

where the matrix R is the unique nonnegative solution, with the maximal eigenvalue strictly less than one, of the matrix equation

$$\lambda I + R(T' - T_C^0 - \lambda I) + R^2 T_C^0 = 0. \quad (B.5)$$

For some values of L and S' , it can be shown that R is of a lower triangular structure (see Feldman and Claybough [18]). In general, however, this is not true and the solution to the above equation is obtained using the iteration

$$R(n+1) = -(R(n)^2 T_C^0 + \lambda I)(T' - T_C^0 - \lambda I)^{-1}, \quad n = 0, 1, 2, \dots,$$

with $R(0) = 0$ so that $\lim_{n \rightarrow \infty} R(n) = R$. (These claims can be routinely verified using the results given in [13 - 17].) Equating the rates of up- and down-crossings from the compound state $E_k = \{(i, j), k \geq i \geq 0,$

$S \geq j \geq 0$ }, for some $k \geq K-1$, we get (see [19]),

$$\lambda x_{Ck} e = x_{Ck+1} T_{Ck}^0 e, \quad k \geq K-1.$$

From (B.4) we have

$$x_{Ck+1} = x_k R, \quad k \geq K-1.$$

Then from the above two equations we get

$$RT_{Ck}^0 e = \lambda e,$$

which can be used as an accuracy check for R .

Now to guarantee the existence of the steady state probability x we need

$$\frac{\lambda}{\mu} < C - \sum_{j=0}^{S'} j \pi_j,$$

where

$$\pi_0 = \left[\sum_{j=0}^{S'} \left(\frac{\lambda_0}{\mu_0} \right)^j \frac{1}{j!} \right]^{-1},$$

$$\pi_j = \left(\frac{\lambda_0}{\mu_0} \right)^j \frac{1}{j!} \pi_0, \quad j = 1, 2, \dots, S',$$

and

$$\pi_j = 0, j = S'+1, S'+2, \dots, S$$

is the solution to $(\pi_0, \pi_1, \dots, \pi_S)T' = 0$ and $\sum_{j=0}^S \pi_j = 1$. The above result is easily verified using the results given in [13 - 17].

Now, from equation (B.2), we get

$$\tilde{x}_{i-1} = \tilde{x}_{K-1} R_{i-1}, i = 1, 2, \dots, K-1, \quad (B.6)$$

where

$$R_{i-1} = -\frac{1}{\lambda} \{ R_i (T_i - T_i^0 - \lambda I) + R_{i+1} T_{i+1}^0 \}, i = 1, 2, \dots, K-1,$$

$$R_{K-1} = I \text{ and } R_K = R.$$

Then from (B.1) we get, after substituting (B.6) for \tilde{x}_0 and \tilde{x}_1 ,

$$\tilde{x}_{K-1} (R_0 (T_0 - \lambda I) + R_1 T_1^0) = 0.$$

This, along with the normalizing condition $\tilde{x}_e = 1$, and after substituting for \tilde{x}_i , results in

$$\tilde{x}_{K-1} \left(\sum_{i=0}^{K-1} R_i + R(I-R)^{-1} \right) e = 1,$$

which yields the solution for \tilde{x}_{K-1} and hence $\tilde{x}_i, i \geq 0$.

B.4 Performance of the Flow Control Scheme

The performance of an integrated multiplexer can be characterized in terms of the following measures: (1) the probability of loss, P_L , for voice, and (2) the mean number of data packets in the system, N . In this section we compute these performance measures based on the queueing model to examine the performance of the flow control scheme.

Since an incoming voice call is lost if the number of voice calls in service is greater than $S-1$, or greater than $S'-1$ and the number of data in the system is greater than $L-1$, we have

$$\begin{aligned}
 P_L &= \sum_{i=0}^{\infty} x_{iS} + \sum_{i=L}^{\infty} \sum_{j=S'}^{S-1} x_{ij} \\
 &= \sum_{i=0}^{\infty} \sum_{j=S'}^S x_{ij} - \sum_{i=0}^{L-1} \sum_{j=S'}^{S-1} x_{ij} .
 \end{aligned} \tag{B.7}$$

Let $\chi = \sum_{i=0}^{\infty} \chi_i \triangleq (y_0, y_1, y_2, \dots, y_S)$. Then from (B.4) we get

$$\chi = \sum_{i=0}^{K-1} \chi_i + \chi_{K-1} R(I-R)^{-1} . \tag{B.8}$$

Then from (B.7) and (B.8),

$$P_L = \sum_{j=S'}^S y_j - \sum_{i=0}^{L-1} \sum_{j=S'}^{S-1} x_{ij} . \tag{B.9}$$

Now the mean number N , of data packets in the system is given by

$$\begin{aligned}
 N &= \sum_{i=0}^{\infty} i x_i e \\
 &= \sum_{i=1}^{K-1} i x_i e + x_{K-1} (I-R)^{-1} R ((I-R)^{-1} + K-1) e,
 \end{aligned}
 \tag{B.10}$$

which is obtained from (B.4)

Numerical Example

Here we consider the special case $S' = S-1$. In this example, the following parameter values are assumed:

$$\lambda = .3, \mu = .1, \lambda_0 = .02, \mu_0 = .01, \text{ and } C = 5.$$

The probability of loss for voice, P_ℓ , and the mean data queue size N for various values of S and L are shown in Tables (B.1 - B.5). A plot of voice loss probability P_ℓ vs. mean data queue size N for $S = 1$ and 2 and various values of L is shown in Figure (B.1).

B.5 Summary and Conclusions

In this paper, we have proposed a priority cutoff flow control scheme for movable boundary integrated voice-data multiplexers. A continuous-time queueing model for the integrated multiplexer with flow control was considered. An efficient numerical solution procedure for the queueing model was developed and the performance measures were computed. A numerical example was presented to illustrate the advantage of the flow control scheme. These results show that the proposed flow control scheme provides a continuum of system operating points and, in fact, performs better than the integrated multiplexing techniques considered thus far.

B.6 References

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L CUTOFF POINT	P_L VOICE LOSS PROBABILITY	N MEAN DATA QUEUE SIZE
0	1	3.3542
1	0.9715	3.3610
2	0.9047	3.3826
3	0.8274	3.4204
4	0.7661	3.4706
5	0.7279	3.5302
6	0.7071	3.6015
7	0.6940	3.6687
8	0.6854	3.7270
9	0.6796	3.7754
10	0.6756	3.8145
11	0.6728	3.8455
12	0.6710	3.8697
13	0.6696	3.8885
14	0.6687	3.9028
15	0.6681	3.9138
∞	0.6666	3.9457

TABLE B.1: $S = 1$

L CUTOFF POINT	P_L VOICE LOSS PROBABILITY	N MEAN DATA QUEUE SIZE
0	0.6667	3.9457
1	0.6531	3.9491
2	0.6159	3.9665
3	0.5651	4.0088
4	0.5198	4.0847
5	0.4897	4.2016
6	0.4685	4.3317
7	0.4529	4.4619
8	0.4412	4.5861
9	0.4322	4.7012
10	0.4254	4.8060
11	0.4201	4.9000
12	0.4159	4.9835
13	0.4126	5.0570
14	0.4101	5.1213
15	0.4080	5.1772
∞	0.4000	5.4942

TABLE B.2: $S = 2$

L CUTOFF POINT	P_L VOICE LOSS PROBABILITY	N MEAN DATA QUEUE SIZE
0	0.4000	5.4942
1	0.3939	5.4981
2	0.3762	5.5186
3	0.3514	5.5758
4	0.3298	5.6896
5	0.3117	5.8403
6	0.2968	6.0125
7	0.2845	6.1960
8	0.2744	6.3842
9	0.2658	6.5728
10	0.2586	6.7590
11	0.2525	6.9409
12	0.2473	7.1171
13	0.2428	7.2867
14	0.2389	7.4493
15	0.2356	7.6044
∞	0.2106	9.7451

TABLE B.3: $S = 3$

L CUTOFF POINT	P_L VOICE LOSS PROBABILITY	N MEAN DATA QUEUE SIZE
0	0.2106	9.7451
1	0.2086	9.7496
2	0.2031	9.7730
3	0.1961	9.8365
4	0.1886	9.9416
5	0.1813	10.0818
6	0.1746	10.2492
7	0.1684	10.4368
8	0.1629	10.6391
9	0.1579	10.8522
10	0.1533	11.0731
11	0.1492	11.2996
12	0.1455	11.5299
13	0.1422	11.7627
14	0.1391	11.9969
15	0.1363	12.2316
∞	0.0956	20.8345

TABLE B.4: $S = 4$

L CUTOFF POINT	P_L VOICE LOSS PROBABILITY	N MEAN DATA QUEUE SIZE
0	0.0956	20.8345
1	0.0852	20.8387
2	0.0942	20.8568
3	0.0928	20.8973
4	0.0910	20.9634
5	0.0892	21.0543
6	0.0872	21.1680
7	0.0853	21.3006
8	0.0835	21.4491
9	0.0817	21.6108
10	0.0800	21.7840
11	0.0784	21.9670
12	0.0769	22.1586
13	0.0754	22.3578
14	0.0741	22.5637
15	0.0732	22.7760
∞	0.0373	38.3271

TABLE B.5: $S = 5$

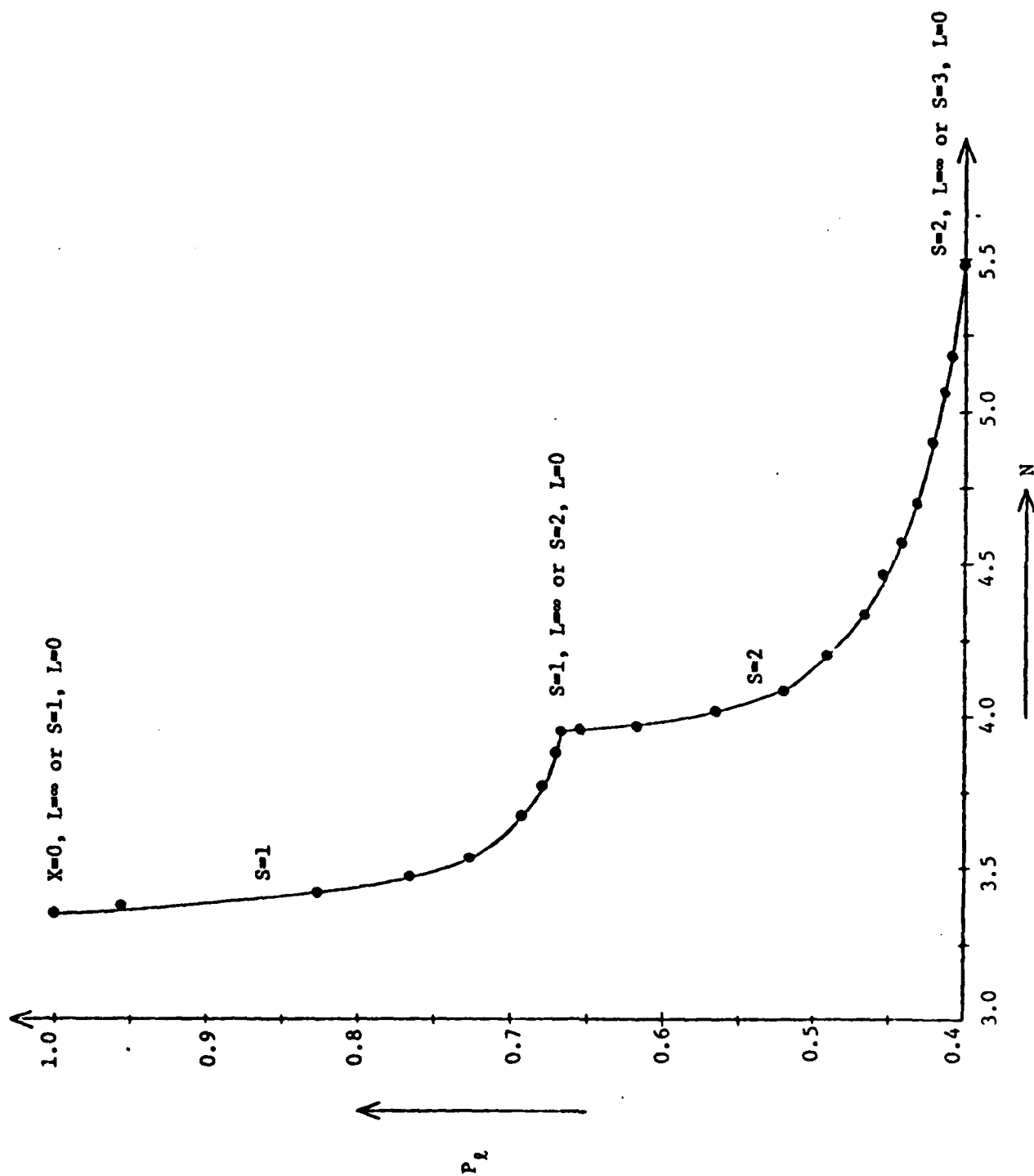


Fig. B.1 A plot of voice loss probability, P_L vs. mean data queue size for $S=1$ and 2 and various values of L .

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